Flow induced excitation on basic shape structures

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Abstract

The study of flow-induced excitation on structures and obstacles is one of the main topics of fluid dynamics related to the practical interests in a large number of engineering applications e.g. aerodynamic, mechanical, civil, naval, etc. New design and project techniques have offered hazardous solutions, resulting in structures that are even more slender and flexible. This has led to a number of situations of self-excited vibration due to the interaction between flow fields and structures. Forces coming from this mechanism depend upon both the incoming flow and the structure motion, giving rise to a strong non-conservative force field, which may eventually lead to a growing structure motion. The aim of this chapter is to offer an overture about the phenomenon of the fluid–structure interaction. Because of the importance that the cylindrical and spherical shapes have in the practical applications and the generalizations that these shapes allow, in this chapter the fluid–structure interaction is mainly referred to these basic shapes.

1 Introduction

Flow-induced excitations of bodies, obstacles and structures in steady or unsteady flows, are at present both a relevant field of research as well as the subject of important studies of theoretical and experimental nature.

International literature reports several studies and contributions relating to such topics for the quasi two-dimensional systems and are summarized in the works of Sarpkaya [1], Ramberg & Griffin [2], Bearman [3], as well as in the papers of Blevins [4] and Naudascher & Rockwell [5].

From the 1970s up to the 1980s, the research was mainly focused on the study and analysis of flow fields and vortex structures generated downstream of
the bodies. The emphasis of the results was aimed at a clear definition of the kinetic characteristics of the currents, related also to the different geometries of the flow field, through a range of values of several dimensionless parameters governing the process, such as Reynold’s number, Strouhal’s number and Keulegan-Carpenter’s number (Bearman [6], Keulegan & Carpenter [7], Sarpkaya [1]).

Subsequently, from the 1980s on, the development of new acquisition and visualization techniques for describing flow field structures, as well as the increase in computational capacities for data processing, allowed the research to study by implementing physical experiments the direct assessment of the effects induced by the flow fields on the bodies (Blackburn & Henderson [8], Lin et al [9], Sheridan et al [10]) and the dynamic responses of the obstacle. In these studies the body is thought as a boundary condition for the flow field.

Only in the last few years, the description of the “interaction” between body and flow presents a different approach. It is focused on the possibility of explaining the different behaviors of bodies in water, and in fluids in general, by looking at the system as a whole. From this point of view, the body, thought of as a “structural system”, does not represent only one of the boundary conditions for the flow field but, due to its geometrical and mechanical characteristics, plays a relevant role in governing the dynamics of the process as well.

In order to point out the main active phenomena in the flow–structure interaction processes, the present chapter deals with the analysis of the sources of excitation acting on the structure, both external and self-excitation, and the dynamic response of the obstacle.

2 The source of excitation – kinematic implications of flow structure on induced excitation

2.1 Fluid–structure interaction

The immersion of a solid body in a turbulent flow induces distortions that are connected to strong kinematic and dynamic instability. The fluid dynamic forces, due to the fluid–structure interaction, can be analyzed in terms of mean and instantaneous components; the latter is responsible for the excitation of vibrations.

According to the dominant excitation mechanism involved (Naudascher [11]) the sources of such vibrations can be classified into four groups (fig. 1): (EIE) Extraneously Induced Excitation caused by fluctuating velocities or pressures which are independent of any flow instabilities originating from the structure and from structural motion, with the exception of added-mass effect; (IIE) Instability-Induced Excitation caused by an instability of the flow due to the presence of the structure; (MIE) Movement-Induced Excitation due to fluctuating forces arising from movements of the vibrating structure; (EFO) Excitation due to Fluid Oscillation caused by a fluid oscillator becoming excited in one of its natural modes. In any of the first three cases (EIE), (IIE) (MIE), the exciting forces may or may not be affected by the simultaneous excitation due to fluid oscillation (EFO). However, even if the sources of excitation are usually studied according to the above classifications, the
Excitation of flow-induced vibrations in a real system is very often complex, since EIE, IIE, MIE and EOF may occur simultaneously.

In fig. 1, the Extraneously Induced Excitation (EIE) is represented by the instabilities of the incoming flow due to its turbulent level, which is not affected by the characteristic of the structure. Other sources of EIE are: earthquakes, machines and machine parts, two-phase flow and oscillating flow. The Instability-Induced Excitation (IIE) is depicted by the vortex shedding downstream to a stationary circular cylinder; the shape of the obstacle mainly affects the kinematic characteristics of the wake. In the case of the Movement-Induced Excitation (MIE), the obstacle is not stationary; its movements interact with the vortex shedding evolution or induce vortex shedding. In this situation the structure behaves like a body oscillator (see Section 3): the transverse movements of the structure induces distortions on the flow field which in turn induces the self-excitation of the structure. Finally, the Excitation due to Fluid Oscillation (EFO) mechanism is represented by standing gravity waves generated between a long pier and the walls of a flume. Flow-induced excitation can be enhanced by the EFO mechanism especially if one of EFO frequencies assumes the natural body-oscillator frequency, the dominant frequency of flow instability, or both (refer to the relevant literature for an extensive discussion on the effects of fluid oscillations: Guilmineau & Queutey [12], Lam & Dai [13], Yan [14]).

The structure of an external flow around an immersed body and the way in which the flow can be described and analyzed often depends on the geometry of the body. Three main categories of bodies are usually considered: (a) two-dimensional objects (infinitely long and of constant cross-sectional size and shape); (b) axisymmetrical bodies (formed by rotating their cross-sectional shape around the axis of symmetry), and (c) three-dimensional bodies that may or may not be symmetrical. In practice there can be no truly two-dimensional bodies, however, many objects are sufficiently long so that the end effects of considering the body as two-dimensional are negligible.

Another classification of body shapes can be made depending on whether the body is streamlined or blunt. Flow characteristics depend strongly on the amount of streamlining present. In general, streamlined bodies (e.g. airfoils, racing cars, etc.) have a little effect on the surrounding fluid in comparison with the effect of blunt bodies (e.g. buildings, parachutes, etc.).
The geometrical shape of structures or vehicles is very complex and the fluid-dynamic efficiency of the shape is usually studied employing a physical model.

Even though the fluid–structure interaction mechanism has been extensively studied on basic shapes, it still presents open questions; therefore a large part of current studies still concern basic shapes (cylinders, prisms, spheres, etc.), also because the fluid-dynamic studies on complex structures are hardly extendible to other shapes or boundary conditions.

The cylinder is one of the basic shapes most studied because of the simplicity of its form and because this form mimics a large number of practical applications. Long-span bridges, tall buildings, tall towers, cables, and so on, are examples of flexible cylindrical structures that are very sensitive to vortex-induced vibrations. The characteristic elongation in the transverse direction makes the cylindrical shapes very sensitive to the induced excitation of the flow. To be able to find the appropriate countermeasures required to control the fluid-dynamic response, the generation mechanism of this response should be clarified first. This mechanism is the flow pattern around the obstacle. Because of the complexity of the flow pattern, in order to understand the fluid–structure interaction mechanism, usually basic cross-sections such as 2-D rectangular cylinders, H-shaped cylinders and circular cylinders have been investigated and mainly in smooth flows.

In smooth steady flow conditions the cross-sectional dimensions of a stationary cylinder are generally the main characteristics responsible for the flow pattern deformations and consequently for the induced excitation on the cylinder. The sensitivity of the flow pattern on the main parameters that affect the phenomenon (such as Reynolds number, turbulent intensity, aspect ratio etc.) depends on the cross-sectional form of cylinder.

While vortex shedding principally depends on the Reynolds number (fig. 2) for a circular cross-sectional form, the vortex shedding mechanism is more complex for a sharp edge cross-section. For a rectangular sharp edged cross-section, the aspect ratio (L/D) generally represents the main parameter to be taken into account.

The forces that the flow induces on a circular cylinder affected by the flow field structures shown in fig. 2, both on the mean and fluctuating components are highlighted in figs. 3 and 4. The figures detail the drag coefficient and the Strouhal number of a circular cylinder versus Reynolds number. The different values of Strouhal number are justified by the characteristics of the boundary layer on the cylinder and of the near wake. In a subcritical range of Re (150 – 300 < Re < 1 × 10^5 – 1.3 × 10^5), the near wake passing a stationary smooth cylinder is laminar, the vortices shed periodically and the force fluctuations correspond to a spectrum of extremely narrow bandwidth. In the post critical range of Reynolds number (1 × 10^5 – 1.3 × 10^5 < Re < 3.5 × 10^6), the boundary layer on the cylinder becomes turbulent downstream from a laminar separation bubble, and the near-wake becomes less regular; the force spectra in this post critical range are rather broadband. In the transcritical range Re > 3.5 × 10^6, finally, the boundary layer becomes turbulent upstream of separation and there is an apparent return of well-defined periodicity of both vortex shedding and force fluctuation with Sh ≈ 0.3 (fig. 4c).
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Figure 2: Flow patterns for flow past a smooth circular cylinder at various Reynolds numbers: (a) $Re \approx 1.5 \times 10^{-1}$, no separation; (b) $Re \approx 1.5 \times 10^{1}$, steady separation bubble; (c) $Re \approx 1.5 \times 10^{2}$, oscillating Karman vortex street wake; (d) $Re \approx 2.5 \times 10^{4}$, laminar boundary layer with wide turbulent wake; (e) $Re \approx 3.2 \times 10^{5}$, turbulent boundary layer with narrow turbulent wake (Munson et al [15]).

2.2 Sharp-edged rectangular cylinder

In the case of a sharp-edged elongated rectangular cylinder, the flow detaches on the upstream (primary separation) and downstream corners (secondary separation), and the flow distortion is affected by several parameters. The vortices generated close to the body develop and shed from it, creating an unsteady wake. The characteristics of the wake are dependent on the Reynolds number of the obstacle $Re = U_0 D / \nu$, but the aspect ratio (L/D) and the incidence angle between cylinder and flow have the main influence on the vortex shedding and therefore on the structural excitation. Figure 5 shows the mean flow field around a rectangular cylinder (L/D = 3) in unbounded flow, numerically obtained by Yu & Kareem [16] at $Re = 1 \times 10^{5}$.

The primary separation of the shear layer occurs on the leading edge. The separated flow initially diverges from the body, with an angle dependent on the separation pressure, and then curves toward the cylinder surface. When the reattachment occurs, the flow makes a region of recirculation known as “separation

Figure 3: Drag coefficient as a function of Reynolds number for smooth circular cylinder and smooth sphere (Munson et al [15]).
bubble”. In such conditions the secondary separation at the downstream edge of the cylinder causes the roll up of the flow in the rear face generating a secondary vortex that periodically sheds from the body surface. In some cases, when the reattachment of the primary separation is unsteady, the two turbulent structures interact and the vortex shedding becomes more complex. To simplify the description of the phenomenon, the main vortex shedding regimes have been defined and classified on the basis of the characteristics of the main vortices involved. In the case of steady flow conditions and rigid obstacle, Naudascher & Wang [17] give the following classification:

LEVS (Leading-Edge Vortex Shedding): the flow separation occurs at the leading-edge with formation of vortices dominating the near wake of the body (fig. 6a);

TEVS (Trailing-Edge Vortex Shedding): a decisive flow separation at the trailing-edge occurs and vortex-shedding is analogous to the von Kármán street behind circular cylinders (fig. 6c);

ILEV (Impinging Leading-Edge Vortices): a flow separation at the leading-edge and impingement of the leading-edge vortices at the side surfaces and/or edges of the body are present (fig. 6b);

Figure 4: Strouhal number, Sh, of vortex shedding (a) and spectra of the lift force component (b, c) from a stationary, smooth circular cylinder in low-turbulence cross flow (Naudascher [11]).

Figure 5: Mean flow field numerically obtained in unbounded flow around a rectangular cylinder, L/D = 3, Re = 1 × 10^5 (Yu & Kareem [16]).
AEVS (Alternate-Edge Vortex Shedding): both the leading-edge and the trailing-
edge mechanisms are present (fig. 6d).

Each vortex type allows a specific dynamical state. Under the flow conditions
above mentioned and for a wide range of Reynolds numbers, the aspect ratio (L/D)
approaches the vortex shedding and the loading on the structure significantly. When
L/D < 2 (fig. 6a), only the primary separation occurs because the shear layer sep-
arates at the leading edge and involves the whole side of the cylinder (LEVS); in
this range of L/D Bearman & Trueman [19] observed that the formation of the
vortex close to the cylinder enlarges the drag coefficient of the obstacle (C_D).
The minimum distance between rear cylinder face and vortex formation occurs for
L/D = 0.64, which corresponds to the maximum value of C_D.

When L/D > 6 (fig. 6c), the flow separated at the leading edge reattaches per-
manently; consequently the trailing edge separation (TEVS) dominates the vortex
shedding. For L/D ≈ 2.8 (fig. 6b), the literature indicates a complex situation of pos-
sible unstable reattachment (Shimada & Ishiara [20], Yu & Kareem [16]). When
α ≠ 0 (fig. 6d), the symmetry of the flow structure is compromised. Consequently,
on one the upper side of the cylinder LEVS prevails and on the lower, TEVS. When
this condition occurs, the vortex shedding is characterized by the AEVS regime.

The vortex shedding behavior is well described by the Strouhal number
(Sh = f_0 D / U_0, where f_0 is the dominant frequency of the vortex shedding).

The chart in fig. 7 reports the Sh obtained by several Authors with rectangular
cylinders of various L/D immersed in unbounded flows. In fig. 7, two discontinuities
of Sh values are evident. The first occurs when L/D is approximately equal to 2.8,
the second occurs when L/D = 6. At L/D ≈ 2.8, the flow pattern is bounded between
the flow separation type LEVS and the flow reattachment type ILEV. The data
dispersion in fig. 7 and the presence of more than one dominant frequency for a
specific aspect ratio mainly depend on the upstream flow characteristics. At the latter
critical aspect ratio L/D = 6, the flow pattern is bounded between the unsteady flow
reattachment type ILEV and the completely steady flow reattachment type TEVS.

For a wide range of conditions, several studies show the negligible influence
of Reynolds number when it exceeds the value Re = 1 × 10^4. On the contrary, the
influence of Re is not negligible for Re < 1 × 10^4 (Okajima [21]). In fig. 8, the Sh
versus Re is reported with an aspect ratio L/D = 3.
Figure 7: Variations of Strouhal number according to L/D ratio for rectangular cylinders in unbounded flow (Shimada & Ishiara [20]).

The free stream turbulent level of the flow passing a circular cylinder, \( T_u = u'_{rms}/U_0 \) (where \( u'_{rms} \) is the standard deviation of the inflow velocity on \( x \) direction), significantly affects the flow pattern and the excitation induced on the cylinder. As shown in fig. 9a, turbulence decreases \( C_D \) at subcritical Re and

Figure 8: Variation of Strouhal number with Reynolds number for rectangular cylinders with L/D = 3 (Okajima [21]).
Figure 9: Effect of the free-stream turbulence, $Tu$, on (a) mean drag coefficient, $C_D$, (b) and on Strouhal number, $Sh$, for a smooth circular cylinder (Naudascher [11]).

increases it in the supercritical range. The rise in value of Strouhal number in the transition range (fig. 4a) occurs at smaller $Re$ as $Tu$ increases (fig. 9b).

In the case of a sharp-edged rectangular cylinder, free stream turbulence level ($Tu$) has received a great attention in literature because it significantly influences the structure and the development of the shear layer separated off the upstream corners (Haan et al [22], Lin & Melbourne [23], Noda & Nakayama [24], Saathoff & Melbourne [25]). The main effect of $Tu$ is to shift the reattachment point. An increase of $Tu$ leads to a progressive shortening of bubble formation and, thus, to a possible strong modification of vortex shedding (Nakamura et al [26]). Noda & Nakayama [24] observed that turbulence shakes the shear layer over a distance comparable with the turbulence scale. The main effects of turbulence occur when $L/D$ is in a range of values near the critical value $L/D = 2.8$. In this range of $L/D$, the reattachment of the leading edge separation is not stable. In this situation, the turbulent inflow with the length scale of the same order as $D$ acts by moving the position of the separated shear layer off the downstream corners, promoting the reattachment.

The behavior of vortex shedding is significantly affected also by the presence of boundaries that limit the evolution of the wake. The presence of boundaries are relevant in a large number of civil applications (e.g. buildings, bridges, pipelines, etc.)

The study of boundary effects has principally been considered in aerodynamic applications especially in terms of blockage ratio ($\gamma_b$), defined as the ratio between the frontal area of the body and the cross-section of the flow without obstacle.

Both for a circular or rectangular cylinder, significant changes in $Sh$ values can occur when the flow confinements are changed. In general, the increasing of the blockage induces an acceleration of the flow near the object, which locally increases the flow velocity (solid blockage) and increases the energy losses in the wake and in the boundary layer (wake blockage). For bluff bodies, the effects of blockage (solid and wake) can be very remarkable and its influence changes the values of
both force coefficients and Strouhal number. These effects are usually taken into account using the following expressions:

\[ C_{F_c} = C_F (1 - \gamma_b)^{n_{CF}} \quad \text{Sh}_c = \text{Sh} (1 - \gamma_b)^{n_{sh}} \]

where \( C_{F_c} \) and \( \text{Sh}_c \) are the corrected force coefficient and Strouhal number, respectively and \( n_{CF} \) and \( n_{sh} \) are experimental coefficients \((0 \leq n_{CF}, n_{sh} \leq 1)\).

The presence of a significant asymmetry of the boundary conditions has also remarkable effects on the structure excitation. These effects are summarized in fig. 10, where the Strouhal number of a circular cylinder is plotted against the elevation ratio, \( G/D \), of the cylinder above a wall (\( G \) is the elevation above the wall and \( D \) is the diameter of circular cylinder). The effects on the cylinder excitation are emphasized by low Re values and affected by the boundary layer thickness (\( \delta \)) above the wall.

The influence of a solid surface on the dynamic effects for a rectangular cylinder has been less considered in literature. A recent study (Cigada et al [33]) highlights that for a rectangular cylinder with aspect ratio \( L/D = 3 \), the presence of a solid surface significantly affects both the force coefficients and the vortex shedding even if the cylinder is placed at relevant elevation, \( G \), from the surface. The solid boundary affects the lift coefficient, \( C_L \), up to \( G/D \approx 3.5 \); in the range \( 3.5 \geq G/D \geq 1 \), \( C_L \) decreases toward the negative value \( C_L = -1 \) then increases up to \( C_L = 1 \) in the range \( 1 \geq G/D \geq 0 \). The influence on drag coefficient seems limited in the range \( 1 \geq G/D \geq 0 \), where \( C_D \) decreases from its typical unbounded value up to \( C_D = 0.7 \).
In hydraulic applications the influence of a free surface is usually considered as one of the main parameters in modelling structure excitation, relevant in the study and assessment of the vulnerability of river bridges under partial or total submergence conditions. Experimental studies, carried out by Denson [35] and Malavasi & Guadagnini [34], highlight the significant influence of the free surface in terms of force coefficients. The influence of the free surface on the flow field structure around the obstacle was provided in Malavasi et al [36].

Figure 11 shows the behavior of $C_D$ versus $h^*$ for different values of the Froude number, $Fr_s$, for a rectangular cylinder with $L/D = 3$ placed at an elevation $G/D = 2.33$ from the bottom of a hydraulic channel, where $C_D$ is computed as:

$$C_D = \frac{F_D}{0.5\rho U_0^2 D},$$

$h^*$ is the dimensionless parameter of the cylinder submersion:

$$h^* = \frac{h - G}{D},$$

Figure 12: Lift coefficient versus $h^*$ with $Fr_s = 0.26$, together with the reference value of $C_L = 0$ corresponding to the unbounded flow condition (Malavasi et al [36]).
and Fr_s is the Froude number referred to the cylinder thickness:

$$Fr_s = \frac{U_0}{\sqrt{gD}}. \quad (4)$$

As $h^*$ increases toward 1, $C_D$ increases independently from Fr_s, with further increases of $h^*, C_D$ reaches its peak value then decreases and tends to an asymptotic value. The peak of $C_D$ seems to depend on the observed value of Froude number and $h^*$.

The free surface also drastically influences the lift coefficient, as shown in fig. 12 where $C_L$ is plotted versus $h^*$ ($Fr_s = 0.26$). The lift coefficient presents a negative peak for $h^* = 1$, after which the absolute value of $C_L$ increases tending to zero (unbounded flow condition) as $h^*$ increases.

In fig. 13, the vortex shedding frequency in terms of Strouhal number and calculated by the frequency analysis of the lift component on the rectangular cylinder is plotted versus $h^*$ for different Fr_s. The significant difference between the ex-

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**Figure 13:** Strouhal number, Sh, versus $h^*$ (G/D = 2.33).

**Figure 14:** Mean flow field reconstruction by velocity field measured around the cylinder with Fr_s = 0.26, G/D = 2.33 and $h^* = 1.0$ (a) and $h^* = 1.4$ (b) (Malavasi et al [36]).
perimental values and the reference value of the unbounded condition may be explained in the features of the confinement of the flow. As shown for example in figs. 14a and 14b, the upper confinement of the flow interferes with the leading-edge separation, changing the structure and the characteristic of the vortex shedding. The asymmetry imposed by the free surface limits the separation on the topside of the cylinder, thus the lack of equilibrium on the vertical loading direction induces significant variation in the $C_L$ value from $C_L = -9$ to $C_L = -2$ as shown in fig. 12.

3 Dynamic response of the structure

3.1 Basic equations

The analysis of the interaction between flow and structure may also be put forth using the behavior of the body as a reference point for characterizing the processes. This allows us to evaluate the dynamic response of the oscillating obstacle, compared to vortex-induced vibrations phenomenon and the main characteristics of the flow field.

The equation of motion generally used to represent the vortex-induced vibrations of a body oscillator, in steady and unsteady flows, is proposed as follows:

$$m\ddot{x} + B\dot{x} + Cx = F(t),$$ (5)

where $x$ is the displacement of the body towards the main flow or in a transversal direction (fig. 15a), $m$ is the total structural oscillating mass, $B$ is the structural damping, $C$ is the spring constant and $F$ is the acting fluid force. In this case, the body oscillator is treated as a discrete-mass system free to vibrate in one/two directions and the fluid force assumes a sinusoidal form:

$$F = F_0 \sin(\omega_s t + \varphi_s),$$ (6)

where $\omega_s = 2\pi f_s$ is the circular frequency and $f_s$ is the frequency of fluid force.

The solution of eqn. (5) is composed by the solution of the homogenous equation given by:

$$x = e^{-\zeta \omega_n t} x_0 \cos(\omega_d t - \varphi) \quad \text{with} \quad \omega_d = \omega_n \sqrt{1 - \zeta^2},$$ (7)

and the particular solution:

$$x = x_0 \cos(\omega_s t - \varphi),$$ (8)

where $\omega_n = 2\pi f_n$ is the natural circular frequency and $f_n$ is the natural frequency of the system.

The frequency $f_n$, can be generally calculated under the hypothesis of perfect joint taking into consideration the contribution of the added mass according to the following relationship:
Figure 15: (a) Simple body oscillator with linear damping (i.e. resistance proportional to velocity); (b, c) histograms of responses for an underdamped ($\zeta < 1$) and an overdamped ($\zeta \geq 1$) case (Naudascher & Rockwell [5]).

$$f_n = \frac{1}{2\pi} \sqrt{\frac{C + C'}{m + m_a}},$$

(9)

where $C'$ represents the added stiffness, which is usually included in the spring constant of the system and $m_a$ the related added mass.

The damping factor or damping ratio, $\zeta$, is defined as:

$$\zeta = \frac{B}{2m\omega_n} = \frac{B}{2\sqrt{(m + m_a)C}}.$$  

(10)

For the underdamped case ($\zeta < 1$) the damping factor can be calculated by the exponentially decaying response for the initial condition $t = 0$, $x = x_0$ and $\varphi = 0$ (fig. 15b), as mentioned by Naudascher & Rockwell [5]. In the case $0 < \zeta < 1$ the coefficient has been obtained through the following equation:

$$\phi = \ln \frac{x_n}{x_{n+1}} = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}},$$

(11)

where $\phi$ is called the logarithmic decrement.

For $\zeta \geq 1$, the displaced body simply returns to its equilibrium position in an exponential fashion (fig. 15c). The damping for the limit case of $\zeta = 1$ is called critical damping.

Since the solution eqn. (7) dies out with time on account of damping, only the steady-state solution eqn. (8) is of general interest. Its frequency is equal to the forcing frequency $f_s = \omega_s/2\pi$ and the amplitude $x_0$ is obtained as:

$$x_0 = \frac{F_0/C}{\sqrt{1 - (\omega_s/\omega_n)^2}^2 + (2\zeta\omega_s/\omega_n)^2},$$

(12)

The phase angle $\varphi$ by which the response $x$ lags the exciting force $F$ is as follows:

$$\tan \varphi = \frac{2\zeta\omega_s/\omega_n}{1 - (\omega_s/\omega_n)^2},$$

(13)

where $\omega_n = \sqrt{C/m + m_a}$ is the natural frequency of the undamped system.
For a body with one torsional degree of freedom, eqn. (5) takes the form:

\[ I_\theta \ddot{\theta} + B_\theta \dot{\theta} + C_\theta \theta = M(t) \]  

(14)

where \( I_\theta \) is the mass moment of inertia of the body, \( M(t) \) is the exciting moment or torque, \( B_\theta \dot{\theta} \) is the damping moment, \( C_\theta \theta \) is the restoring moment, \( \zeta_\theta = B_\theta / 2I_\theta \omega_n \) is the damping ratio and \( \omega_{\theta n} = \sqrt{C_\theta / I_\theta} \) is the undamped circular natural frequency.

The response to a harmonic exciting moment \( M(t) = M_0 \cos \omega_s t \) is:

\[ \theta = \theta_0 \cos(\omega_s t - \varphi) \]  

(15)

where \( \theta_0 \) and \( \varphi \) are the amplitude of torsional vibration and the phase angle respectively.

### 3.2 Dynamic response in resonance conditions

In the study of flow-induced vibrations a condition of particular interest is the resonance phenomenon when the vortex shedding frequency is close to the natural frequency of the structure. This occurs because the resonance phenomenon generates critical conditions in the structures in terms of stability and structural stress corresponding to potential collapsing of the structures themselves.

Another case of relevant importance is one in which the frequency of the body oscillations matches the frequency of the wake vortex. In such cases the processes are outlined as lock-in or synchronization. The body tends to “pulse” presenting large amplitude and the system, even if it does not assume the resonance condition, is subject to relevant stress. In such conditions the oscillation amplitudes, transversal to the fluid flow (y-direction), are always found to be much larger than streamwise motions (x-direction).

Studies on the analysis of the vibrating structures nearing the conditions of resonance in bounded and free surface flows, have highlighted the existence of a strong dependence of the maximum transverse amplitude \( A_{\max}^* \) on some non-dimensional groups, as shown below:

\[ A_{\max}^* = A^* \left( \frac{U_0}{f_n D}; S_G; \frac{h}{D} \right) \]  

(16)

where \( A_{\max}^* = y_0 / D \) is the ratio between the maximum transverse amplitude and the characteristic dimension \( D \) of the body, the ratio \( \frac{U_0}{f_n D} \) is the reduced velocity \( U^* \), \( h / D \) is the ratio between the water depth and the characteristic size of the body and \( S_G \) is the Skop-Griffin parameter defined as follows:

\[ S_G = 2\pi^3 Sh^2 (m^* \zeta), \]  

(17)

where \( Sh \) is the Strouhal number \( \frac{U_0}{f_n D} \).

Concerning the influence of the body shape, the \( S_G \) parameter takes into account both boundary conditions of the oscillating body, through \( m^* \) (ratio between the structural mass \( m \) and the added mass \( m_a \)) and \( \zeta \), as well as the flow induced force.
through Sh. Under resonance conditions, the Strouhal number is assumed constant, thus the maximum transverse amplitude, $A^{*}_{max}$, depends on the Skop-Griffin parameter and, as seen in eqn. (17), on the combined mass-damping parameter $m^{*} \zeta$.

Figure 16 summarizes the results of several experiments for different values of $m^{*}$ in terms of maximum transverse amplitude, $A^{*}_{max}$, versus $m^{*} \zeta$. From this figure, it does not seem possible to make a “singular” curve of $A^{*}_{max}$ versus $m^{*} \zeta$. Sarpkaya [1] originally stated that a simple observation of the motion equation immediately shows that the response of the system is independently governed by mass and damping. By analyzing three pairs of low-amplitude response data, each pair of them at similar values of $m^{*} \zeta$ but different $m^{*}$ values, he observed a large influence of mass ratio on $A^{*}_{max}$. In fact Sarpkaya [43] states: one should use the combined parameter $m^{*} \zeta$ only for $m^{*} \zeta > 0.40$ while for $m^{*} \zeta < 0.40$ the dynamic response of system is governed by $m^{*}$ and $\zeta$ independently.

Khalak & Williamson [41] carried on a set of experiments over a wide range of $m^{*}$ ($m^{*} = 1 \div 20$) under the same experimental conditions showing that even for low $m^{*}$ of the order 2 and very low mass-damping down to the value $m^{*} \zeta \sim 0.006$, the use of a single combined mass-damping parameter collapses peak amplitude data very well, even for a wide independent variation of parameters $m^{*}$ and $\zeta$ (fig. 16). In this way they extended the value of $m^{*} \zeta$ proposed by Sarpkaya by two orders of magnitude.

Furthermore, in the case of elastically mounted systems, they observed two different types of response depending on the high or low combined mass-damping parameter $m^{*} \zeta$. In fact for low $m^{*} \zeta$ values, there are three different branches of response: the initial, the upper and the lower ones which present two jumps in
the magnitude of oscillating displacement (fig. 17). They found that the transition between the “initial” and “upper” branch was hysteretic, while the transition from the “upper” to “lower” branch involved an intermittent switching.

On the contrary, for high values of combined mass-damping parameter $m^*\zeta$, Feng [37] observed only two branches of response: the initial branch and the lower one. The passage between the two branches, as can be seen in fig. 17, occurs with a jump and the body reaches conditions of resonance.

Furthermore, Govardhan & Williamson [42], by visualization techniques (Digital Particle Image Velocimetry), showed that the change from the initial branch to the upper one, depends on the jump in the angle phase between the force induced by the shedding of the main vortex and the displacement of the body (fig. 18). This jump is characterized by a change in the form of the vortex wake downstream of the body by a mode “2S”, indicating 2 single vortices shed per cycle, to mode “2P”, meaning 2 pairs of vortices per cycle (fig. 18). Under this condition the value of the body oscillating frequency, $f$, passes across the natural frequency in water generating a resonance phenomenon. On the other hand, the passage from the upper branch to the lower one is characterized by the presence of a phase-difference between the total fluid force and the displacement of the body which tends to go toward a periodic uniform trend. In such cases no change in the form of the wake is observed.

For high values of $m^*\zeta$ the passage from the initial branch to the lower one depends on the jump of a phase between both the force components, the total force and the force induced by the vortex and such jump is related to a change in the form of the wake.

Referring to fig. 17, the behavior found for three-dimensional structures, with elementary geometrical forms (ex. spheres), is sensitively different from that observed for two-dimensional structures. In fact, the data of Jauvtis et al [44] relating to the oscillations of a sphere, indicate the presence of two distinct modes of

![Figure 17: Maximum amplitude versus reduced velocity for different bodies: Khalak & Williamson [41] and Feng [37] on the cylinders; Jauvtis et al [44] and Mirauda & Greco [45, 46] on the spheres.](image)
response. The first mode of response (Mode I) is manifested in the presence of resonance conditions, when the frequency of the shedding of the vortex is close to the natural frequency of the body, and a synchronization regime is observed between the force and the response. When the average velocity of the flow increases, the system shows the presence of periodic oscillations characterized by high values of displacement that represent the second mode of response (Mode II).
In fig. 17, data from Mirauda & Greco [45, 46] are also reported. The first set (squares), referring to a steel sphere in a free surface flow with a high value in the combined mass-damping parameter, is characterized by low oscillations and show only the initial branch without a jump in amplitude and, therefore, they do not exhibit hysteresis phenomena. The second series (triangles), characterized by values of \( m^*\zeta \) lower than the previous ones, are close to the first mode of response. It outlines how the system tends to reach the resonance conditions where vortex-shedding frequency is equal to the natural frequency.

The results reported in fig. 17 can be better outlined by referring to figs. 19 and 20 which report the values of the \( f^* \), ratio between the body oscillating frequency \( f \) and the body natural frequency \( f_n \), versus the reduced velocity \( U^* \). In particular fig. 19 shows the data observed by Khalak & Williamson [41] for vibrating circular cylinders with mass ratio equal to 2.4, 10.3, and 20.6 and fig. 20 the data observed by Jauvtis et al [44] and Mirauda & Greco [45, 46] for vibrating spheres with a mass ratio equal to 80, 7.9 and 1.14, respectively.

In the figures, the horizontal line represents the condition in which the oscillating frequency \( f \) is equal to the natural frequency and the diagonal line is the condition in which \( f \) is equal to the vortex-shedding frequency for the static cylinder.

It has been observed that for low mass ratios, oscillation frequency starts from the natural frequency as the velocity \( U^* \) increases and this transition is characterized by the presence of hysteresis.

On the contrary, in the case of high mass ratios the synchronization regime decreases and the value of \( f^* \) remains close to the unity for all values of \( U^* \).

This is true both for the two-dimensional structures (cylinders) as well as for three-dimensional structures (spheres).
In the case of free surface flow, the dynamic response is also conditioned by the parameter h/D. In fact, fig. 21 shows the experimental results of Mirauda & Greco [45, 46] for different values of relative submergence and for a limited range of reduced velocity ($U^* = 0.98 \div 8$).

In this range, it is possible to observe how the relative submergence influences the dynamic response of the system, the frequency ratio $f^*$ increases with h/D.
This behavior can be shown through the effect that the deformation of the free surface has on the oscillations of the sphere. In fact, for values of $h/D = 1$ the free surface deforms and the vortex layer is generated between the free surface and the upper obstacle surface. This layer gives rise to a near-wake conditioning the frequency body response. Vortex generation selects frequency ranges, which can include the “proper” obstacle frequency and can involve typical aspects related to the locking-in effects. Values of $h/D > 1$, on the other hand, pull the system away from the condition of lock-in and synchronization.

4 Conclusion

Flow induced excitations on structures represent a relevant and topic related to several modern theoretical and practical engineering problems. The aim of this chapter was to provide updated information about findings concerning two aspects related to the excitation on vibrating structures. In fact, the approach followed takes into account two main points of view on the processes: firstly, the flow field and the effect due to the turbulent features of the wake have been discussed as the source of the vibration on the structure. Secondly, the interaction between flow and structure has been proposed in terms of dynamic response of the obstacle. The process of flow induced excitations focuses on the framework of cause-effect referring to basic-shape structures like circular, and/or rectangular cylinders and spheres.

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List of symbols

\[ B = \text{coefficient of mechanical damping} \ [MT^{-1}] \]
\[ B_\theta = \text{torsional damping coefficient} \ [ML^2T^{-1}] \]
\[ C = \text{spring constant coefficient} \ [MT^{-2}] \]
\[ C' = \text{added stiffness} \ [MT^{-2}] \]
\[ C_A = \text{potential added mass coefficient} \ [/] \]
\[ C_D = \text{drag coefficients} \ [/] \]
\[ C_L = \text{lift coefficient} \ [/] \]
\[ C_\theta = \text{torsional spring constant} \ [ML^2T^{-2}] \]
\( D \) = characteristic dimension of body \([L]\)

\( f \) = oscillating frequency of the body \([T^{-1}]\)

\( f^* \) = frequency ratio \((f/f_n)[/]\)

\( f_n \) = natural frequency of body oscillator \([T^{-1}]\)

\( f_0 \) = frequency of vortex shedding \([T^{-1}]\)

\( f_s \) = forcing frequency \([T^{-1}]\)

\( F \) = fluid force on an obstacle \([MLT^{-2}]\)

\( F_0 \) = force amplitude \([MLT^{-2}]\)

\( F_D \) = unit per length drag force \([MT^{-2}]\)

\( Fr_s \) = Froude number referred to the cylinder thickness \([/]\)

\( G \) = elevation of the obstacle by the wall \([L]\)

\( h \) = depth of water \([L]\)

\( h^* \) = dimensionless parameter of obstacle submersion \([/]\)

\( h/D \) = relative submergence \([/]\)

\( I_\theta \) = mass moment of inertia \([ML^2]\)

\( L \) = length of body along the flow direction \([L]\)

\( m \) = mass of body \([M]\)

\( m^* \) = mass ratio \([/]\)

\( m_a \) = added mass \([M]\)

\( M \) = moment \([ML^2T^{-2}]\)

\( M_0 \) = moment amplitude \([ML^2T^{-2}]\)

\( Re \) = Reynolds number \([/]\)

\( S_G \) = Skop-Griffin parameter \([/]\)

\( Sh \) = Strouhal number \([/]\)

\( t \) = time \([T]\)

\( Tu \) = free stream turbulent level \([/]\)

\( u'_{rms} \) = standard deviation of \(U_0\) on inflow main direction \([LT^{-1}]\)

\( U^* \) = normalized velocity \([/]\)

\( U_0 \) = mean velocity of the incoming flow \([LT^{-1}]\)

\( U_w \) = average velocity in the wake \([LT^{-1}]\)

\( x, y \) = stream-wise, transverse displacement \([L]\)

\( x_0, y_0 \) = vibration amplitude in \(x,y\) direction \([L]\)

\( y \) = transverse displacement \([L]\)

\( y_0 \) = vibration amplitude in \(y\) direction \([L]\)

\( \delta \) = boundary layer thickness \([L]\)

\( \gamma_b \) = blockage ratio \([/]\)
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\[ \mu = \text{fluid dynamic viscosity } [ML^{-1}T^{-1}] \]
\[ \nu = \text{fluid kinematic viscosity } [L^2T^{-1}] \]
\[ \rho = \text{fluid density } [ML^{-3}] \]
\[ \zeta = \text{damping ratio } [/] \]
\[ \zeta_\theta = \text{torsional damping ratio } [/] \]
\[ \theta = \text{angular or torsional displacement } [/] \]
\[ \theta_\theta = \text{amplitude of torsional vibration } [/] \]
\[ \varphi = \text{phase angle } [/] \]
\[ \phi = \text{logarithmic decrement of mechanical damping } [/] \]
\[ \omega = \text{circular frequency } [T^{-1}] \]
\[ \omega_d = \text{circular frequency of damped oscillator } [T^{-1}] \]
\[ \omega_n = \text{circular natural frequency } [T^{-1}] \]
\[ \omega_s = \text{circular forcing frequency } [T^{-1}] \]
\[ \omega_{\theta n} = \text{circular natural frequency of torsional vibration } [T^{-1}] \]

References

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